**AEE 342: Aerodynamics, Project 2b – Analysis of Non-symmetric Airfoil Flows**

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There are many different flow conditions that can be simulated and observed around an airfoil, but certain flows are of particular interest when evaluating its flight performance. Namely, the superposition of uniform rectilinear flow (URF) with vortex flow is largely responsible for the generation of lift and drag, some major indicators of airfoil performance. However, other types of flows are typically modelled over airfoils in order to satisfy some conditions, or to obtain certain data. In this investigation, the goal is to compute the pressure distribution over different airfoils at different angles of attack through a superposition of URF, source flow, and vortex flow. Several different ways to define these flows under conditions consistent with theory are evaluated, as too are the assumptions necessary for such conclusions. The analysis to follow will aim to properly formulate the problems under consideration, to model them on a theoretical basis, and to evaluate the assumptions made in the process.

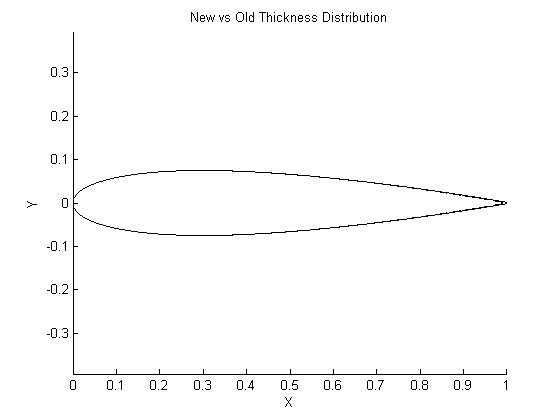
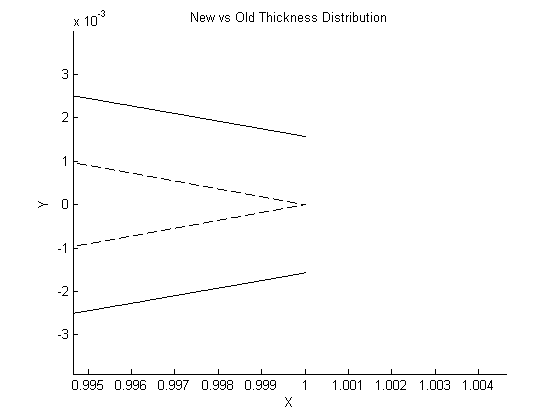
The first task is to prepare the geometry of the airfoil so that it can best be shown to satisfy physical principles. Namely, this refers to the Kutta Condition which can be represented well on an airfoil with a wedge-shaped trailing edge. The given thickness distribution for a NACA 4 digit airfoil describes an airfoil with a trailing edge that is blunt and does not end precisely at x = 1. Several modifications can be made to enforce this condition. Since it is desired that the surface of the airfoil be differentiable, it would be best to modify the equation itself so that it behaves as desired. The original thickness distribution is given by

Where is the maximum thickness of the airfoil as a percentage of chord length. In order to come to a point at the trailing edge, the thickness must define a point at the coordinate (1, 0). Plugging this condition into the thickness distribution yields

And further

From here, it is clear that one of the coefficients must be altered so that this condition is made true. Since x will always be less than 1, the best coefficient to choose would be the fifth one because it would have the lowest overall weight, particularly at low values of x. Therefore

Thus yielding the new thickness distribution

These two distributions are plotted below for comparison. The two figures are of the same data, but the one on the right is scaled so that the distinction between the two airfoils at the trailing edge can be exaggerated. The solid line is of the old data and the dashed one is of the new. Visually, it is clear that the new thickness distribution satisfies the posed requirement and does not appear to deviate substantially from the definition of the NACA airfoil. This is reaffirmed by some quantitative analyses used to measure the two airfoils. The two geometries consistently had the same maximum thickness and the point was nearly always located at the same value of x. Only after plotting the geometry that would correspond to a system of 10000 panels did the index of the maximum thickness shift by one point. For a hypothetical airfoil with a chord length of 1 meter, this would shift the point of maximum thickness by only 0.2 mm. This result falls within a very reasonable margin of error, especially for the purposes of this investigation.

With the geometry prepared, the vortices may be superimposed into the flow. First, a single point vortex may be placed. The best place to apply this single vortex would be at . Since vortex components are derived in polar coordinates, it is best to place the vortex at the leading edge in order to simplify the conversion to a Cartesian coordinate system. After this, a simple shift of coordinates can be performed to move the vortex wherever is desired. Generally, for multiple vortices, the positioning of each should maintain a consistent spacing and be located along the camber line. This is so that the vorticity distribution is kept relatively consistent. Additionally, the vortices should be placed near the midsection and leading edge of the airfoil, since this is where the greatest strength is needed. For this reason, 4 vortices can be placed at 0.1c, 0.2c, 0.3c, and at 0.4c to account for these general tendencies of airfoils and the flow around them. A fewer or greater number of vortices can be applied with a similar distribution. If a vortex sheet is to be used, then a panel method should be adopted to discretize the strengths along the airfoil surface. This will generally yield a better solution because it can be used to constrain the flow based on the geometry of the airfoil. Additionally, it will provide a much smoother distribution of vortices and allow for the easy computation of several other useful values such as the velocity along the surface of the airfoil. However, there are some other changes that will make the implementations of these two methods quite different.

One very important part of the implementation of flows around a surface is the set of geometry integrals. These define the nature of the interaction between the flow and the airfoil surface by introducing it as an impenetrable object. This condition is enforced by making all velocities normal to the airfoil surface equal to zero. Specifically, it is applied by taking the partial derivative of the component normal to the surface of the velocity potential, yielding the velocity normal to the surface. For the superposition of URF with source flow as found in the source panel method, this is represented as

By adding vortex flow to the system, the equation becomes

This is the superposition of the three flows satisfying the geometry and calculating the velocity everywhere. A very similar equation would be used for a point vortex by adding a single value instead of a sum.

Next, it is necessary to